The Relationship between Gerrymandering Classification and Voter Incentives

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Abstract

Gerrymandering is the process of drawing electoral district maps in order to manipulate the outcomes of elections. Increasingly, computers are involved in both drawing biased districts and attempts to measure and regulate this practice. The most high-profile proposals to measure partisan gerrymandering use past voting data to classify a map as gerrymandered (or not). Prior work studies the ability of these metrics to detect gerrymandering, but does not explore how the metrics could affect voter behavior or be circumvented via strategic voting. We show that using past voting data for this classification can affect strategyproofness by introducing a game which models the iterative sequence of voting and redrawing districts under regulation that bans outlier maps. In experiments, we show that a heuristic can find strategies for this game including on real North Carolin maps and voting data. Finally, we address questions from a recent US Supreme Court case that relate to our model. This is a summary of "Meddling Metrics: the Effects of Measuring and Constraining Partisan Gerrymandering on Voter Incentives" appearing in EC2020

1 Introduction

Algorithms, machine learning, and automated systems have become crucial players in the game of drawing and evaluating US electoral districts. Governments have increasingly utilized software to draw districts that influence the outcomes of elections (e.g., to favor a particular political party, incumbent politician, or racial group). Conversely, academics and enthusiastic citizens have proposed algorithms that purport to be fair alternatives [Liu *et al.*, 2016; Cohen-Addad *et al.*, 2018; Ryan and Smith,], a familiar promise of technology that does not always hold true even with the best of intentions. More recently, major US court cases have highlighted computational approaches to evaluate whether or not a district map is gerrymandered to favor a particular political party [Chikina *et al.*, 2017; Herschlag *et al.*, 2018; Cho and Liu, 2018; Cho, 2019]. These legal challenges include a Pennsylvania Supreme Court case which led to the redrawing of congressional districts in that state [LWV, League of Women Voters of Pennsylvania v Commonwealth of Pennsylvania No 159 MM 2018] and a recent landmark US Supreme court case [Cas, Rucho v Common Cause No 18 422 588 US 2019].

That US Supreme Court ruling essentially left it to states to decide how to address the issue, if at all. In doing so, state governments will need to consider the downstream effects of how they choose to measure and regulate partisan gerrymandering. To that end, we initiate the study of how classifying district maps using past voting data can incentivize strategic voting to circumvent regulations. We show that careful scrutiny should be given to any measurement which uses voting data to evaluate and affect the choice of district maps.

1.1 Districting and gerrymandering definitions

For the sake of brevity, we restrict our discussion to the United States political system, where gerrymandering has become a highly contentious issue, as a guiding example. In US politics, many government representatives are elected via single-member district plurality systems. Voters are partitioned into districts which each elect representatives with a plurality vote. Election districts may be redrawn every 10 years following a population census for reasons such as balancing the number of voters per district.

Historically, state governments have been charged with drawing district maps. The majority party in a state's government controls the redistricting process, and this is the system we model in our work. More recently, some states have transitioned to a system where an "independent" commission draws the maps. We note that the problem of how to measure gerrymandering remains relevant in this alternate approach, but we do not explicitly model this nascent system.

Gerrymandering is the process of drawing electoral district maps to manipulate the outcomes of elections. This can be done by *packing* many voters from one group into a single district where they cast more votes than needed to win, or *cracking* voters from a group into multiple districts where they cast votes for losing candidates in each district. In both packing and cracking, the idea is to make one group "waste" as many votes as possible while another group wastes fewer votes. The goal depends on the groups being targeted and intended outcome. *Partisan gerrymandering* favors one political party over another. Similarly, *racial gerrymandering*

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supports or disadvantages defined racial groups. *Incumbent gerrymandering* is slightly different in that it creates a bias toward re-electing an incumbent candidate. We focus on partisan gerrymandering. However, these different notions are often entangled such as when there is a correlation between racial demographics and party membership or when partisan gerrymandering creates "safe districts" where incumbents have an advantage. In fact, laws that prevent the cracking of racial groups have been used to justify packing members of a racial group for the apparent purpose of partisan gerrymandering as in Florida's famously snake-like District 5.

The space of legal district maps is restricted by a set of (sometimes competing) constraints and objectives that vary from state-to-state. The most common restrictions are contiguity, community integrity, population balance, hole-freeness, and compactness. *Contiguity* means that districts should be contiguous spaces although they may only be connected by narrow paths. *Community integrity* states that districts should avoid splitting defined communities (e.g. counties, towns, etc.) if possible. However, communities are routinely split ostensibly to meet other objectives. *Population balance* is the objective that the number of voters in each district should be as balanced as possible in order to give roughly the same weight to each person's vote. Congressional district populations within a state vary as much as 897,080 in Texas District 22 to 713,480 in Texas District 13 [Bureau, 2017][1]. Between states, the difference can be much larger: 1,050,493 in Montana's at-large district versus 520,389 in Rhode Island's District 2 [Bureau, 2017]. *Hole-freeness* states that no district should be completely surrounded by one other district. Finally, *compactness* is perhaps the least consistently or easily defined goal with many possible definitions. These include k-median-like objectives minimizing the average distance a voter has to travel to a center point in their district and objectives minimizing ratio of area to perimeter.

1.2 Computer science combating gerrymandering

Two main directions where computer science has become involved in combating partisan gerrymandering are classifying a given map as unfairly gerrymandered (or not) and drawing fair maps. Crucial to both is the question of how to measure gerrymandering and define fairness in this context.

Unfortunately, it is not simple to measure whether partisan gerrymandering has occurred nor is it straightforward to say that partisan gerrymandering is "unfair" in the US legal context (e.g., unconstitutional). While gerrymandering may seem obvious just by looking at a map, this "eyeball test" is not robust [Duchin, 2018]. A strange looking map may accurately conform to the geography of a state (features such as mountains or highways that perturb distances between voters), while a reasonable looking map may be gerrymandered. Then, supposing we have some test to show that a map is gerrymandered, we must further show that the practice we measure violates the law in some way. In the US, gerrymandering by itself is not illegal even though candidates choosing their voters instead of the other way around may violate sensibilities. The US Supreme Court has ruled that racial gerrymandering is unconstitutional while incumbent gerrymandering is allowed [Cas, Thornburg v Gingles No 83 1968 478 US 30 1986; Cas, Shaw v Reno No 92 357 509 US 630 1993].

Another seemingly obvious standard which uses past voting data is *proportionality*, the idea that the proportion of seats assigned to each party should be close to the proportion of votes received. However, drawing a map which achieves proportional representation is impossible for states like Massachusetts [Duchin *et al.*, 2018]. More importantly in the US, courts have rejected proportionality tests.

More central to our work is the recent study of outlier maps. An *outlier map* is one which is abnormal by some measure of representation such as seat count awarded to each party based on past voting data. A map may be an outlier if it awards 3 seats to a given party while almost every other legal map awards 4 or 5 seats. In practice, the space of all possible maps is too large to check. So the state-of-the-art is to approximately randomly sample from the set of all legal maps using Markov Chain Monte Carlo techniques [Chikina *et al.*, 2017; Herschlag *et al.*, 2018; Cho and Liu, 2018]. One can generate thousands of random maps and use past voting data to determine how many seats each party would win on each map. Comparing a given map to this random sample determines if it is an outlier. The seat count measure of representation we consider is used in [Herschlag *et al.*, 2018]. Other works use more fine-grained measures to capture smaller changes in the distribution of voters to districts [Chikina *et al.*, 2017].

1.3 Other related work

The classic Gibbard-Satterthwaite Theorem [Gibbard and others, 1973; Satterthwaite, 1975] established that at least one of the following must hold in ordinal voting systems electing a single candidate: (1) Dictatorship (one voter chooses the winner); (2) Only two candidates; or (3) The system is not strategyproof and inspires tactical voting. However, our result shows strategizing in two party elections with single candidates elected per district, in a model where a vote in one district can affect the outcomes of multiple districts in future elections. Thus, we highlight a situation where the Gibbard-Satterthwaite Theorem may appear to apply, but does not.

The disparity of different groups having different abilities to strategize was studied in [Hu *et al.*, 2019]. We show how measuring fairness creates additional opportunities for strategic manipulation in voting systems and open the question of whether additional disparity follows. This is relevant in elections as political parties leverage such disparities to disenfranchise groups of voters (e.g., using voter id laws to target people in urban areas without drivers' licenses or restricting early voting to target people with less flexible working hours).

2 Our contributions

We illustrate how computational efforts to detect gerrymandering using past voting data can incentivize voters to strategize and vote untruthfully. This shows that a single-member district plurality system under the policy of banning outlier maps with only two parties is not strategyproof. A party holding the majority of seats can vote and draw districts strategi-

[1] This Texas population data is from 2016 estimates while the current Texas map was adopted in 2013 based on 2010 census data.

cally to increase the number of seats they win. To further explore this phenomenon, we implement a heuristic for identifying pure strategies that lead to winning more seats via tactical voting and run experiments on both simple models and real data. Finally, we consider questions from the US Supreme Court case Rucho v. Common Cause [Cas, Rucho v Common Cause No 18 422 588 US 2019] relating to outlier maps. Our observations may also be relevant to the study of election security, voter fraud, and tampering with election results. We reveal a scenario wherein a political party could gain in the long run by generating votes for the opposing party. Such a tactic may be difficult to detect if one is assuming that a cheating party would not reduce its own votes.

3 Modeling the game of voting and redistricting

We introduce a simple game to model the cycle of drawing districts, voting, redrawing districts, and voting again. Drawing districts is constrained by a regulation which uses past voting data to decide if a district map is legal or not. The goal of the model is to clearly illustrate how a regulation can incentivize voters to vote strategically rather than truthfully and better understand how regulations might be circumvented.

In this game, there are two political parties, *red* and *blue*, denoted R and B, respectively. Voters are vertices in a graph $G = (V, E)$ with the number of vertices $|V| = n$ and the edge set E signifying neighboring voters. We define a map m as a partition of G into k districts. Each district must be a connected component in G with size equal to n/k (we assume for simplicity that k is odd and n/k is an odd integer). In this way, we enforce the rules that districts must be contiguous and perfectly balanced in terms of population.

Each voter v has a true preference for one of the two parties. The set of true preferences for all voters $P \in \{R, B\}^n$ is known to both parties at the start of the game and does not change. Having true preferences known to the parties captures the fact that parties may know more about their voters than the regulation which only "sees" how people have voted. Without loss of generality, we assume the red party holds the majority of seats at the start of the game. We use $Q \in \{R, B\}^n$ to denote the set of votes from the most recent election which is known to both parties and the regulation.

Finally, we have a regulation $\psi\colon m \mapsto \{legal, banned\}$ which determines whether a given map m is legal to use or banned and cannot be used. We focus on the regulation of *banning outliers*. In this case, ψ also takes as input the previous votes Q, set of all possible maps M (or a set of maps sampled from M), and a threshold $\tau \in (0, 1]$. If the fraction of maps awarding the same number of seats to the red party as m is less than τ, then m is banned. Otherwise, it is legal.

At the start of the game, we let $Q = P$, assuming that in a prior election, voters cast votes according to their true preferences for two reasons. First, it is easier to analyze, but still addresses our major question of whether strategic voting can be incentivized by gerrymandering regulation. Second, this models the adoption of a new regulation.

The game proceeds in four rounds to observe the effect of voting on which maps can legally be chosen. Each party in this zero-sum game seeks to win as many seats as possible.

Round 1: The majority party (red) draws a map m subject to a gerrymandering regulation ψ. The voters' true preferences are used as the past voting data to regulate this first round (i.e., $Q = P$).

Round 2: Voters vote simultaneously, but voters of the same party may collude. Each district's seat is awarded to the party with the majority of votes.

Round 3: The party which won the majority of seats in Round 2 draws a new map m subject to ψ. However, Q is now the set of votes from Round 2 and this may affect ψ.

Round 4: Again, voters vote simultaneously, but voters of the same party may collude. Each district's seat is awarded to the party with the majority of votes.

While this game only captures two election cycles, we show that it reveals an incentive to vote strategically. Natural extensions to more cycles or more rounds of voting before redistricting could be considered in the future. In this short game, voters will vote their true preferences in Round 4 since these votes only determine the outcome of a single election.

3.1 Simplifying assumptions

To simplify the analysis of collusion, we assume each party controls all of its voters and chooses how they will vote. Furthermore, we require each voter to vote for one of the two parties. They cannot abstain or vote for a third party. To guarantee clear majorities without ties, we use an odd number of districts with an odd number of voters in each district.

We consider the utility of a party to be a linear function of seat count. This reduces the space of strategies to explore since a party cannot gain utility by sacrificing a seat in one round in order to gain a seat in another round. However, one could also envision a more complex model with a nonlinear function that captures real world effects. For example, in the US system, there are often a large added benefits to crossing the thresholds of a simple majority and a two-thirds majority.

In the US system, there are typically multiple elections between the rounds of redistricting that can occur following each decennial population census. Thus, our abstraction replaces a series of elections with a single voting round.

4 A simple example game on a 3×3 grid

To illustrate our game from Section 3 and the effects of regulation, we apply the regulation of banning outliers to a specific set of voter preferences on a 3×3 grid with 3 districts of size 3. Here, we can visualize an exhaustive set of maps.

A 3×3 grid admits 10 maps of 3 districts when contiguity and population balance are the only restrictions. Figure 1 shows all 10 maps along with a set of true voter preferences and strategies. The top row of voters prefer the blue party, while the bottom two rows prefer the red party. We can see plainly in Figure 1 that the red party would prefer the first map which partitions the voters into three columns. This map cracks the blue party so that red wins all 3 seats and is the only map in which red wins 3 seats as opposed to 2.

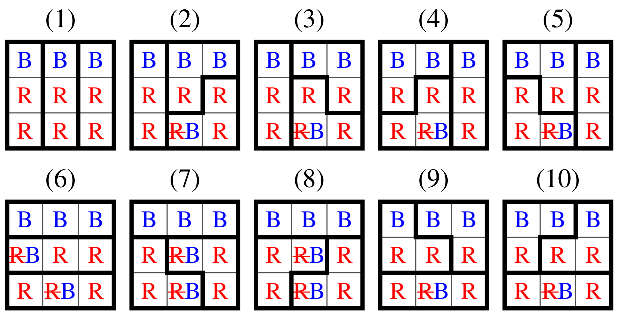

Figure 1: All 10 possible maps of a 3×3 grid partitioned into 3 contiguous districts of equal size. Blue B's indicate voters for the blue party and red R's indicate voters for the red party. Squares with ~~R~~B represent red party voters who can vote for the blue party in Round 2 to make map 1 look like a non-outlier map when drawing districts in Round 3. Only map 1 awards 3 seats to the red party under true preferences, making it the best for red, but also an outlier.

For this simple example game, we employ the regulation of banning outliers with a threshold τ strictly greater than 0.1, the smallest meaningful threshold for this graph (we use smaller, more realistic τ later in our experiments). Thus, the first map in Figure 1 will be banned with respect to the voters' true preferences P and therefore banned in Round 1. Because this is the preferred map for the red party, but it cannot be chosen given the regulation ψ and voting history $Q = P$, we call map 1 the target map for red. This is the map red wants to draw in Round 3 to win an extra seat in the game.

Now, suppose some red voters were to vote for blue in Round 2 without giving up a seat. This could make it appear that the first map will award 1 seat to blue in future elections when in fact the red voters could then vote truthfully to award all seats to red. Blue is allowed to respond with strategic voting, but in this case red has a pure strategy for any choice of starting map that blue cannot respond to (Observation 1).

Observation 1. *For any valid starting map, the red party has a pure strategy (illustrated in Figure 1) for winning 5 seats total during the two voting rounds in the game from Section 3.*

Note that in several maps (2, 3, 4, 5, 9, and 10) in Figure 1, red can flip a single voter to blue in order to make it appear that the middle column district of map 1 will be awarded to blue. This will not cause red to lose a seat in Round 2 voting compared to voting true preferences and it makes map 1 look like a non-outlier according to the voting data from Round 2. To try to counter this, blue could flip its voter in the middle column to red. However, this would cause blue to lose that voter's district in Round 2, immediately giving 3 seats to red. In other maps (6, 7, and 8) blue can afford to flip one voter to red without losing a district and this forces red to flip two voters from two separate districts to guarantee that map 1 will not be an outlier in Round 3. Thus, given any legal starting map (all maps except map 1), red can vote strategically to win 5 seats overall, whereas truthful voting only yields 4 seats.

5 Experiments

In the first experiment, our "state" is a 5×5 grid with 5 districts of size 5. To exhaustively study a 60/40 split in the true preferences of voters, we consider all possible sets of true preferences where the majority party has 15 of 25 total voters ($\sim$3.2 million preference sets). A benefit of using this simple model is that we can consider all 4,006 contiguous and balanced district maps in order to identify outliers for a given set of preferences. This divorces the analysis of outlier regulation policies from the questions of how to sample maps and detect outliers that are the focus of [Chikina *et al.*, 2017; Herschlag *et al.*, 2018; Cho and Liu, 2018]. Our heuristic seeks a pure strategy for voting in Round 2 and a more favorable map to legally choose in Round 3. For banning outliers with a threshold τ of 2%, about half of the true preference sets allow maps awarding all 5 seats to the majority party (no strategizing needed). Among most of the remaining preference sets, the majority party is limited to choosing maps in Round 1 that award fewer than 5 seats under true preferences, but our heuristic is able to find a pure strategy which leads to winning an additional seat in Round 4.

In the second experiment, we use real voting data and maps from North Carolina (a notoriously gerrymandered state) in a restricted model where only majority party voters can vote strategically. Court cases have struck down North Carolina's 2012 and 2016 congressional maps for partisan gerrymandering. To address this issue, the "Beyond Gerrymandering" project sponsored by the Duke Center for Political Leadership, Innovation, and Service brought together an independent commission of 10 bipartisan retired judges to redraw North Carolina's congressional map without the use of past political data or election results to generate a more fair district map known as the *judges' map*. According to past voting data and a sampled set of maps, North Carolina's actual *2016 map* is an outlier awarding 10 out of 13 seats to republicans and this was used in the court case against the map. By contrast, the judges' map awards 9 out of 13 seats to republicans and is not an outlier. However, if an outlier ban prohibits the 2016 map, we find a strategy for choosing the judges' map in Round 1, but voting strategically in Round 2 such that the 2016 map becomes legal in Round 3 (net gain of 1 seat).

6 Rucho v. Common Cause

We briefly address two questions raised during the recent US Supreme Court case Rucho v. Common Cause [Cas, Rucho v Common Cause No 18 422 588 US 2019]. First, skeptics of the outliers metric for gerrymandering classification, including several US Supreme Court justices, have asked whether it is a proxy for a proportionality rule. However, using basic grid models, we can show that for a non-trivial fraction of preference sets, proportional maps would actually be labeled as outliers rather than favored. Second, the notion of individual harm is central to the argument that partisan gerrymandering is unconstitutional. For a given individual, suppose most maps place them in a district where their chosen party wins. Are they entitled to be in such a district? We show that no deterministic redistricting can meet this test with a grid graph example where several voters can be placed in a district where their party wins in over 64% of maps, but no single legal map provides this opportunity to all of them.

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